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THE STABILITY OF A METASTABLE SHOCK WAVE IN A VISCOELASTIC MEDIUM UNDER TWO-DIMENSIONAL PERTURBATIONS†

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The unsteady motions of a viscoelastic medium are considered, taking account of a small anisotropy and a small non-linearity. The behaviour of a metastable, quasi-transverse shock wave when it interacts with non-one-dimensional perturbations is investigated numerically. The stability of this wave under non-one-dimensional perturbations of large amplitude is demonstrated.

It is well known [1–3] that the solutions of the self-similar problem of a piston and the decomposition of an initial discontinuity are non-unique for a wide class of elastic media and initial conditions. In this case, shock waves exist to which a second solution of the problem of the decomposition of a discontinuity corresponds, which consists of a system of shock waves and simple waves moving with different velocities. This means that, generally speaking, the initial shock wave can decompose into the above-mentioned system of waves. We shall call such shock waves metastable shock waves. If they are excluded when constructing the solutions, then the solutions of the problems of elasticity turn out to be unique.

Their instability could serve as a basis for the above-mentioned exclusion of metastable shock waves. The stability of fast shock waves, including metastable shock waves, has been proved in the linear approximation [4]. However, non-linear instability of a shock wave occurs, which manifests itself in the form of their non-linear decomposition when they interact with perturbations. In order to investigate the possibility of such a decomposition, it is necessary to include mechanisms which "spread" discontinuities.

In a Voigt viscoelastic medium, a continuous travelling wave [3] corresponds to each shock wave. This wave is called the shock wave structure [5]. The interaction of the structure of a metastable shock wave with spatially bounded, one-dimensional perturbations with the same orientation as the metastable shock wave in question has been investigated previously [6, 7] using numerical experiments. In this case, the asymptotic forms of the solutions when $t \to \infty$ corresponded to one of the two self-similar solutions of the problem of the decomposition of a discontinuity in the domain of non-uniqueness. It was shown that interaction with a background inhomogeneity or with another one-dimensional wave can lead to the irreversible decomposition of the metastable shock wave such that, after the interaction, a system of waves is generated which corresponds to the second solution of the problem of the decomposition of a discontinuity, mentioned above. It has been revealed that, for decomposition such a to exist it is necessary, first, that the amplitude of the perturbations should be comparable in magnitude with the amplitude of the metastable shock wave and, second, that, after the interaction, the newly formed waves have succeeded in separating to a distance which is greater than the width of the structure of the metastable shock wave in question.

In spite of the fairly rigorous conditions which are necessary in order for decomposition of the metastable shock wave to occur, it remained possible to assume that, during the motion through a random background, a metastable shock wave will sooner or later encounter a sufficiently large perturbation which will destroy it. On account of this, it is of interest to investigate, in the same formulation as earlier in [6, 7], the interaction of a metastable shock wave with non-one-dimensional perturbations.

The results of a numerical solution of two-dimensional viscoelastic problems of the interaction of a metastable shock wave, with fairly slowly changing perturbations, which are periodic with respect to the tangential coordinate, are presented below. The results of the solution of these problems show that,

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if there is a sufficiently large segment of the unperturbed metastable shock wave, then the decomposition of the metastable shock wave into a system of waves is reversible that is, after a time, the solution obtained is restored to the solution that corresponds to the initial metastable shock wave. This indicates the high stability of the metastable shock wave and the possibility of healing the wounds inflicted on it by the strong, but spatially bounded, external actions. These results suggest that metastable shock waves actually exist.

1. ONE-DIMENSIONAL UNSTEADY SOLUTIONS. DESCRIPTION OF THE SOLUTIONS IN THE DOMAIN OF NON-UNIQUENESS

The weakly non-linear quasitransverse waves, which propagate over a homogeneous background in the positive direction of the x axis in the case of a medium of small anisotropy, can also be described by a simplified system of equations, which follows from the system of equations of the non-linear theory of elasticity [3, 8], namely

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x}\left(\frac{\partial R(u_1,u_2)}{\partial u_\alpha}\right)=\nu\frac{\partial^2 u_\alpha}{\partial x^2}$$

$$\alpha=1,2,\quad \nu=\frac{\mu}{2\rho_0} \tag{1.1}$$

$$R(u_1,u_2)=\frac{1}{2}a(u_1^2+u_2^2)+$$

$$+\frac{1}{2}g(u_2^2-u_1^2)-\frac{1}{8}\varkappa(u_1^2+u_2^2)^2 \tag{1.2}$$

Here $g>0$ is the anisotropy parameter, a is the characteristic velocity when there is no non-linearity and anisotropy ($\varkappa=0, g=0$), $\varkappa$ is a constant with the dimension of velocity, which characterizes the non-linear effects and ν is the kinematic coefficient of viscosity. The sign of the elastic constant $\varkappa$ has a substantial effect on the behaviour of the quasitransverse simple waves and the shock wave. The case when $\varkappa>0$ is considered below.

When there is no viscosity ($\nu=0$), system (1.1) is of the hyperbolic type and has two families of characteristics, fast and slow, with velocities $c_1 \leqslant c_2$. The evolutionary shock waves are correspondingly separated into fast and slow shock waves. It has already been mentioned that, when there is no viscosity, the problem of "a piston" has a non-unique solution [2]. Two different systems of self-similar waves can correspond to one and the same initial conditions $u_\alpha=U_\alpha$ (point A in Fig. 1)

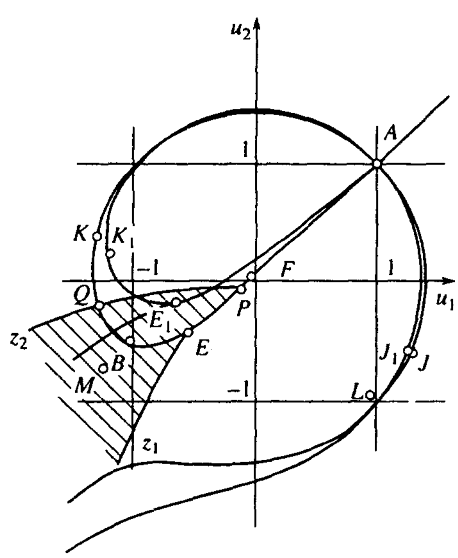

Fig. 1

when $t = 0, x > 0$ and boundary conditions $u_\alpha = u_\alpha^*$ when $x = 0, t > 0$ if the point with coordinates u_1^* and u_2^*, which specify the boundary conditions, belongs to the domain of non-uniqueness (shown hatched in Fig. 1). The geometry of the domain of non-uniqueness depends on the parameters U_1, U_2, g and $\varkappa$.

The numerically constructed shock adiabatic curve $AFPEQKAJ$ of the quasitransverse shock waves with a starting point $A(1, 1)$ for a medium with $\varkappa = 1, g = 0.03$ is shown in Fig. 1. The points of the segments AJ and EK are states behind the evolutionary fast shock waves and the points of the segment AF are states behind the slow shock waves. The domain of non-uniqueness of the solutions (the domain of values of u_α^* for which there are two solutions) is bounded by the segment PE of the shock adiabatic curve with starting point A and, also, by the segment QP of the shock adiabatic curve, constructed from point Q as the starting point, which corresponds to slow shock waves, as well as by the segments EZ_1 and QZ_2 of the integral curves of the simple non-reversing slow waves. The point Q is determined by the following condition: the velocity W of the shock wave $A \to Q$ is equal to the velocity of the shock wave $A \to J$ (if U_1 and U_2 are sufficiently large, $\varkappa(U_1^2 + U_2^2) > 2g$, for example, then such a point exists).

For the media being considered with $\varkappa > 0$, one of the solutions in the domain of non-uniqueness (the solution of the first type) consists of a fast shock wave, which is represented in the u_1, u_2 plane by a jump from point A to a point of the evolutionary segment QE (this is the same metastable shock wave mentioned earlier), and a slow shock wave or simple wave following behind it. The second system (the solution of the second type) when $\varkappa > 0$ contains a "complex" fast wave, consisting of a fast Jouguet shock wave $A \to J$ (the point J is the Jouguet point with respect to a state behind the jump at which $W = C_2^+$), a fast simple wave $J \to L$ and a fast Jouguet shock wave $L \to M$ (the Jouguet condition $W = C_2^-$ is satisfied at the point L, which characterizes a state in front of the discontinuity). A slow shock wave or simple wave propogates behind the complex fast wave at a lower velocity. Such a solution, if it is constructed for a certain point on the segment QE of the shock adiabatic curve, is the system of waves which arise if the corresponding metastable shock wave is decomposed.

2. THE TWO-DIMENSIONAL UNSTEADY MOTIONS OF ELASTIC AND VISCOELASTIC MEDIA

We shall use the simplified systems of equations derived in [4] to describe the two-dimensional unsteady motions of a viscoelastic medium when all the required quantities depend on the time t and the Lagrangian coordinate x and depend only slightly on the Lagrangian coordinate y (in the initial state x and y coincide with the Cartesian coordinates x_3 and x_2 respectively).

$$\frac{\partial u_\alpha}{\partial t} + (f + (-1)^\alpha g)\frac{\partial u_\alpha}{\partial x} - \frac{\varkappa}{2}\frac{\partial}{\partial x}[(u_1^2 + u_2^2)u_\alpha] = \nu\frac{\partial^2 u_\alpha}{\partial x^2} - m\int_{x_0}^{x}\frac{\partial^2 u_\alpha}{\partial y^2}dx \tag{2.1}$$

The left-hand sides of Eqs (2.1) and (1.1) are identical. The first term on the right-hand side of Eqs (2.1) describes the effect of viscosity, the second term on the right-hand side of Eqs (2.1), which takes into account the interaction with respect to y, has the same form as the corresponding terms in the Kadomtsev–Petvriashvili and Khokhlov–Zabolotskaya equations [10].

In the calculations, the value of x in front of the system of waves being studied, where it can be assumed that u_1 and u_2 take the unperturbed constant values, is chosen as x_0.

Equations (2.1) represent the behaviour of the solutions in a system of coordinates moving in the positive direction of the x axis relative to the initial system of coordinates at a constant velocity a–f, where a is the velocity of the characteristics relative to the medium in the linear isotropic approximation which occurs in expression (1.2). The quantity f will subsequently be chosen from considerations of convenience. Here, the linear and non-linear terms on the left-hand side of system (2.1) are found to be of the same order. The coefficients g, $\varkappa$, ν and m can take any values if the units of measurement u_α, x, t and y are dealt with in a suitable manner.

According to the results obtained earlier in [4], the equality $m = a/2$ holds in the initial, untransformed variables. The ratio of the last term (which takes account of the effect of non-one-dimensionality) in Eqs (2.1) to the non-linear term on the left-hand side is equal to $(a/\Delta a)(L_x^2/L_y^2)$ in order of magnitude. Here, Δa denotes the quantity $\varkappa u^2$, which characterizes the change in the velocity of the characteristics accompanying the effect of non-linearity (u is the characteristic size of the quantities u_α and L_x, L_y are the characteristic dimensions with respect to the variables x and y respectively). Here, it has been assumed that $\Delta a/a \ll 1$ (a slight non-linearity) and that $L_x^2/L_y^2 \ll 1$ (closeness to the one-dimensional case). Since the sense of these strong inequalities allows a large degree of arbitrariness, the (last) spatial term in Eqs (2.1) can be both small with respect to the remaining terms as well as quite large.

We will now estimate the order of magnitude of the term responsible for the non-one-dimensionality, starting from the assumption that the viscous and non-linear terms are of the same order of magnitude as the values in the one-dimensional case. We then obtain that the characteristic scale of the change in the quantities with respect to the x axis is equal to $L_x \sim \nu/(\varkappa u^2)$, and the order of magnitude of the non-linear term is determined by the expression $\varkappa^2 u^5/\nu$. The ratio of the "spatial" term to the non-linear term in Eqs (2.1) turns out to be equal in order of magnitude to $M = m\nu^2/(L_y^2 \varkappa^3 u^6)$. The dimensionless parameter M characterizes the intensity of the interaction of the different segments of the wave with one another with respect to the variable y.

Problems are considered below in which $u \sim 1$ and the estimate obtained enables us to compare the results of different versions of the calculation with one another. In particular, a decrease in m is equivalent to an increase in the scale L_y, which characterizes the dependence of the solution on y.

3. DESCRIPTION OF THE NUMERICAL EXPERIMENTS

The results of the numerical solution of a number of initial-boundary-value problems for Eqs (2.1), for which the self-similar solutions described in section 1 represent the asymptotic forms when $t \to \infty$, are presented below.

The equations were written in the form of implicit non-linear difference equations which were linearized using Newton's method and solved by the matrix sweep method [11]. The integrals on the right-hand sides of Eqs (2.1) were calculated by the method of rectangles and the values of the functions $u_1(t, x, y)$ and $u_2(t, x, y)$ in the integrand were assumed to be equal to the values of these functions in each preceding iteration.

The coefficient ν in system (2.1) was chosen in such a way that the term $\nu\partial^2 u_\alpha/\partial x^2$, which describes the physical viscosity, was significantly greater than the computational errors which arise when solving difference equations approximating a differential problem.

The initial-boundary-value problem on the interaction of a metastable shock wave (a wave of the type QE), which is represented by the jump $A \to B$ (Fig. 1), with a two-dimensional perturbation is formulated in the following manner. The system of equations (2.1) is solved in the domain $0 \leqslant x \leqslant l$, $0 \leqslant y \leqslant L$, $t \geqslant 0$. The boundary conditions are specified in the form

$$\begin{aligned} &x = l, y > 0: \ u_\alpha = U_\alpha \ (\text{point } A); \quad x = 0, \ y > 0: \ u_1 = u_\alpha^* \ (\text{point } B) \\ &y = 0, \ \ y = L, \ \ 0 < x < l: \quad \partial u_\alpha/\partial y = 0 \end{aligned} \tag{3.1}$$

The initial conditions ($t = 0, 0 < x < l$) are specified in the form

$$0 < y \leqslant y_1: \ u_\alpha = u_\alpha^{(2)}(x); \quad y_1 < y < L: \quad u_\alpha = u_\alpha^{(1)}(x) \tag{3.2}$$

In connection with the formulation of the boundary conditions, we note that the specification of the boundary conditions when $y = 0$ and $y = k$ in the form of the derivatives with respect to y being equal to zero ensures the possibility of a periodic (with period $2L$) extension of the solution with respect to y. The boundary conditions when $x = L$ correspond to the state in front of the wave, which is a fast wave, so that the state at a sufficient distance ahead of the wave can be assumed to be unperturbed.

In choosing the initial data (the functions $u_\alpha^{(1)}(x)$ and $u_\alpha^{(2)}(x)$), use was made of the results in [7], in which the problem of the interaction of a metastable shock wave with the inhomogeneity of the background was considered in a one-dimensional formulation. In [7], the inhomogeneity of the background was specified by the change in the coefficient g over a certain time interval τ (the time of the interaction between the metastable shock wave and the inhomogeneity). An increase in the parameter g was produced such that, if the magnitude of g did not subsequently change, there were no asymptotic forms containing a metastable shock wave. The formation of asymptotic forms of a second type (that is, the decomposition of the initial metastable shock wave had begun) then occurred at fairly large values of τ and the solution, after giving the initial value to the quantity g, no longer reverted to the initial form. For small values of τ, the structure of the metastable shock wave was restored.

The dependence of the initial data $u_\alpha^{(1)}(x)$ and $u_\alpha^{(2)}(x)$ on x was chosen in the following manner. The steady-state structure of the metastable shock wave, taken from [6], was specified as the initial data $u_\alpha^{(1)}(x)$. The functions $u_\alpha = u_\alpha^{(1)}(x)$, represented by the continuous curves in Fig. 2, correspond to the structure of the metastable shock wave with the state in front of it $u_1 = U_1 = 1$, $u_2 = U_2 = 1$ (point A in Fig. 1) and a state behind the metastable shock wave $u_1 = u_1^* = -1.05$, $u_2 = u_2^* = -0.45$,

$g = g_1 = 0.03$, $\varkappa = 1$, $\nu = 0.024$. The parameter f in Eqs (2.1) was chosen such that, as the time increased, the metastable shock wave did not depart from the computational domain. Functions taken from [7] were chosen as $u_\alpha^{(2)}(x)$. These functions correspond to a certain stage in the one-dimensional decomposition of the same steady metastable shock wave as the result of an increase in g up to a certain value $g = g_2$ with the previous values for the remaining parameters. The shock adiabatic curve for $g_2 = 0.1$ is shown in Fig. 1 (curve $AE_1K_1AJ_1$). The point B when $g_2 = 0.1$ lies in the domain where only one self-similar solution of the second type exists when $\nu = 0$. The functions $u_\alpha^{(2)}(x)$, plots of which are shown in Fig. 2 by the dashed curves represent the previously obtained [7] solutions of the problem of the decomposition of a metastable shock wave at a certain instant of time when the decomposition had already become irreversible.

The initial data (3.2) are specified in such a way that, when $y = y_1$, a change in the initial conditions occurs. In this formulation of the initial conditions, the difference equations poorly approximate the differential equations (2.1) as a consequence of the substantial difference between the functions $u_\alpha^{(1)}(x)$ and $u_\alpha^{(2)}(x)$. In order to remove this shortcoming in the initial conditions, "transition zones" were introduced such that, when $y = y_1$, there was no stepwise change but a gradual change in the type of solution.

The initial data in the range $y_1 \leqslant y \leqslant y_1 + y_p$ (y_p is the width of the transition zone) were specified in the following manner

$$u_\alpha^{(p)}(x) = (1 - f(s))u_\alpha^{(1)}(x) + f(s)u_\alpha^{(2)}(x)$$

$$s = (y - y_1)/y_p, \quad f(s) = s^2(2 - s^2)$$

The function $f(s)$ brings about smoothing. It satisfies the conditions

$$f(0) = 0, \quad f(1) = 1, \quad f'(0) = 0, \quad f'(1) = 0$$

The values of $u_\alpha^{(2)}$ to the left of the domain of rapid change in these functions differ somewhat from the values $u_\alpha^{(1)}$ in the same domain [7]. This difference is due to the fact that, during the decomposition of a metastable shock wave which has been reduced to the functions $u_\alpha^{(2)}(x)$, a slow shock wave departed on the left, which slightly changed the values of u_α and emerged beyond the limits of the computational domain. Only fast waves, which are somewhat distorted by viscosity, are located within the limits of the computational domain. Before carrying out the calculations, it had been specially verified that the arrival of a slow wave on the left-hand boundary ($x = 0$) did not lead to the appearance of any appreciably reflected fast wave (which could distort the process being investigated).

Taking account of the existence of transition zones, the initial conditions (3.2) were written as follows:

$$0 \leqslant y \leqslant y_1: \qquad u_\alpha = u_\alpha^{(2)}(x) \tag{3.3}$$

$$y_1 \leqslant y \leqslant y_1 + y_p: \quad u_\alpha = u_\alpha^{(p)}(x) \tag{3.4}$$

$$(y_1 + y_p < y < L): \quad u_\alpha = u_\alpha^{(1)}(x) \tag{3.5}$$

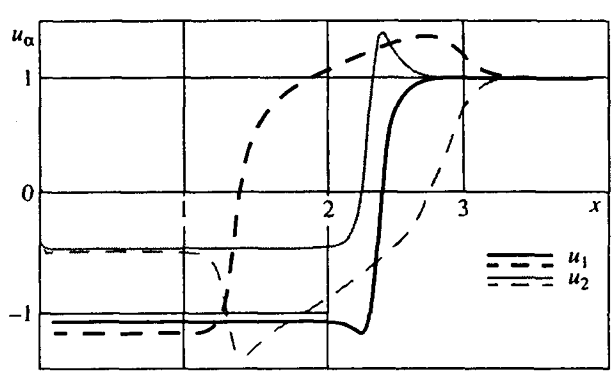

Fig. 2

Test calculations of the initial-boundary-value problem (2.1), (3.1), (3.3)–(3.5) were carried out for different values of the parameter y_p, and it was noted that when y_p was increased (an increase in the thickness of the transition zone) the difference equations were a better approximation of the differential equation. As a result of the test calculations, a value of y_p was determined such that the results of the calculations for a specified step size Δy with respect to the variable y and with a stepsize $\Delta y/2$ are identical within a specified accuracy (0.001).

The calculations of the initial-boundary-value problem (2.1), (3.1), (3.3)–(3.5) enabled as to observe the process of the recovery of the structure of the metastable shock wave. We subsequently describe and demonstrate this process in graphs.

In one of the versions of the calculation, the initial conditions ($t = 0$) were taken in the following form: the initial conditions (3.3) are specified in two successive layers ($y_j = \text{const}$, $y_{j+1} = y_j + \Delta y$, $j = 1, 2$); then ten successive layers ($j = 3, 12$) correspond to the transition zone with initial conditions (3.4) and two layers ($j = 13, 14$) correspond to the initial conditions (3.5) (an unperturbed metastable shock wave). A graph of the function $u_1(x, y)$ when $t = 0$ is shown in Fig. 3. The initial-boundary-value problem which has been formulated was solved for different values of the parameter m.

When $m = 0, y = 2$ (in the system of equations (2.1) there are no terms describing the interaction between layers with respect to the variable y), the following changes occurred when the computation time interval was increased. The asymptotic form of a solution of the second type is gradually formed in the five layers of the intermediate domain, adjacent to the boundary, which represents a solution of the second type. The asymptotic form of a solution of the first type is formed in the remaining layers of the intermediate domain.

For small values of the parameter m ($m = 0.5, 1, 1.3, 2, 3$) ($M = 3.6735 \times 10^{-7}, 7.347 \times 10^{-7}, 1.4694 \times 10^{-6}, 2.2041 \times 10^{-6}$), $\Delta y = 2$, the asymptotic form of the first type is gradually formed in the transition layer ($j = 12$), which is adjacent to the layer ($j = 13$) in which the asymptotic form of the solution of the first type is specified when the computing time interval is increased. An asymptotic form of the first type is subsequently formed in the layer $j = 11$ and so on. The reorganization of the solution in each of the successive layers is accompanied by the emission of a slow wave which propagate's to the left. This slow wave does not interact with the boundary (there are no perturbations reflected to the right). Thus, the asymptotic form of a solution of the first type is formed in the whole of the computational zone.

The evolution of the solution of the initial-boundary-value problem when $m = 30$ ($M = 2.2041 \times 10^{-5}$), $\Delta y = 2$ can be observed in Fig. 4(a–d) ($t = 0.18, 0.6, 1.8, 2.7$), where graphs of the functions $u_1(x, y)$ are plotted for successive instants of time. The start of the formation of the asymptotic form of a solution of the second type in layers $j = 8, 9, 10$ can be seen in Fig. 4(a). The asymptotic form of the solution of the second type is formed in layers $j = 7$–14 in Fig. 4(b). The completion of the formation of the asymptotic form of a solution of the second type can be seen in Fig. 4(c, d). Hence, for $m = 30$ and the existence of the same number of layers which represent the asymptotic form of a solution of the first type and of layers with an asymptotic form of the second type, the asymptotic form of a solution of the second type is formed, that is, the initial metastable shock wave ceases to exist. If the formulation of the initial-boundary-value problem is changed and a number of layers with an asymptotic form of

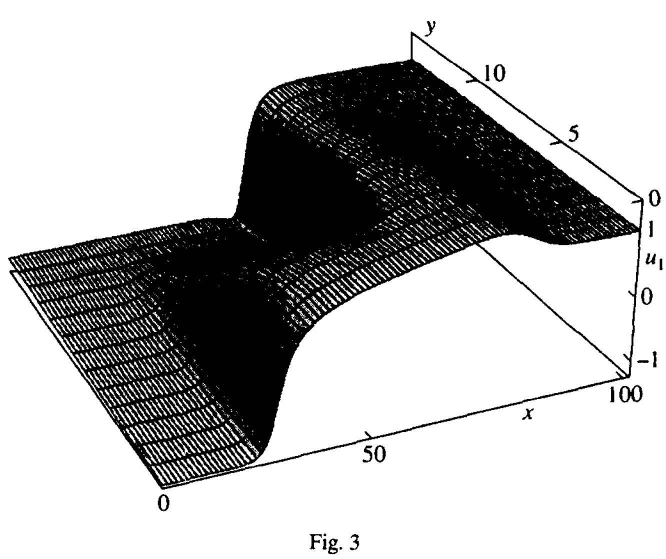

Fig. 3

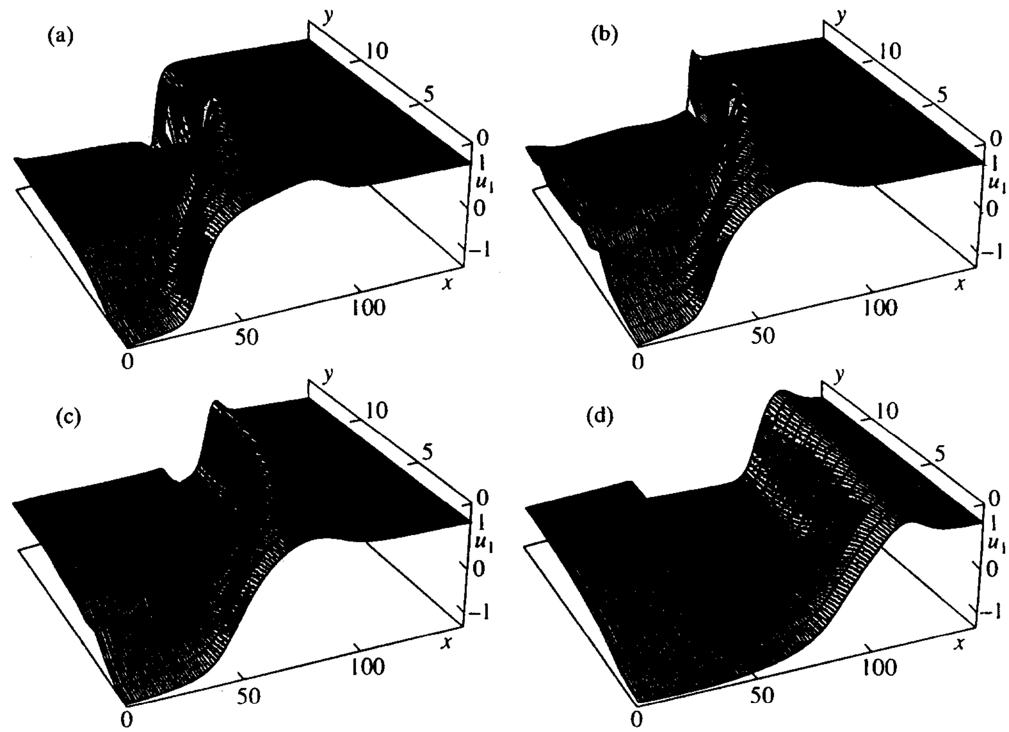

Fig. 4

the first type is specified at the initial instant of time, which is substantially greater than the number of layers with an asymptotic form of the second type and, at the same time, the remaining parameters of the problem remain unchanged, an asymptotic form of the first type is formed in all the layers, that is, the metastable shock wave is restored.

The evolution of the solution of the initial-boundary-value problem having a substantially larger characteristic dimension L than the initial-boundary-value problem in Fig. 4 and a substantially larger parameter $m = 100$ is shown in Fig. 5(a–d). When $t = 0$, the initial conditions (3.3) are specified in four successive layers ($y_j = \text{const}, y_{j+1} = y_j + \Delta y, j = 1, \ldots, 4$); then ten successive layers ($j = 5, 14$) correspond to the transition zone with the initial conditions (3.4), and 21 layers ($j = 15, 35$) correspond to the initial conditions (3.5) (an unperturbed metastable shock wave) and $\Delta y = 2$. The formation of the asymptotic form of the first type (a metastable shock wave, the function $u_1(x, y)$) can be observed in Fig. 5(a–c) ($t = 0.6, 1.2, 2.4$). At the following instant of time ($t = 3.63$, see Fig. 5d), it is clear that the metastable shock wave has been formed for all values of y. The same initial-boundary-value problem was solved when $m = 200, 1000$ ($M = 2.35 \times 10^{-5}, 1.175 \times 10^{-4}$). For these values of the parameter m, the metastable shock wave was restored for all values of y with time. It has therefore been shown that the solution of the problem, when the width of the domain with an unperturbed metastable shock wave is sufficiently large, always leads to restoration of the metastable shock wave for any value of the parameter m.

A similar initial-boundary-value problem was calculated for different values of the parameters g_1, g_2, y_p, a different value of the segment (y_1, y_2) (which characterizes the size of the inhomogeneity), different positions of the initial point A and different positions of the point B, which describes the left-hand boundary condition. In particular, initial-boundary-value problems were investigated for those initial parameters for which the solution of the problem of the interaction between a metastable shock wave and a one-dimensional perturbation [7] showed that the decomposition of this wave is of an irreversible nature, that is, the initial metastable shock wave is not restored with time. The same metastable shock wave was restored in the interaction with a two-dimensional perturbation.

The results of the calculations lead to the conclusion that perturbations which are periodic with respect to a variable along the front of the metastable shock wave (even if this perturbation is of a finite amplitude) do not lead to the decomposition of the metastable shock wave if the scale L, which characterizes the period of the perturbations along the wave front, is large (or the parameter M is small). When the metastable shock wave interacts with isolated perturbations, its restoration can also be

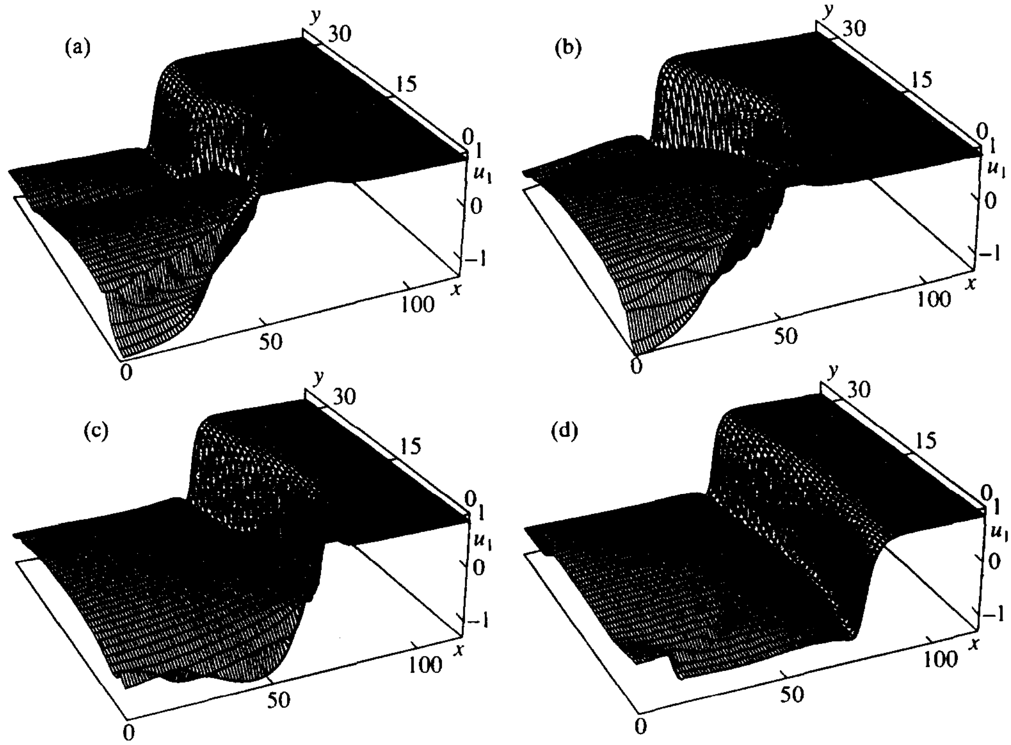

Fig. 5

expected. In the case when L is small (M is a large number), decomposition of the metastable shock wave can occur under certain conditions.

The stability of a metastable shock wave when $\varkappa > 0$ has therefore been demonstrated and, consequently, it is necessary to consider them as existing. This means that the conclusion that the solution of the problem of the decomposition of an arbitrary discontinuity for elastic media is non-unique in the case when viscosity and other factors, which "spread" discontinuities, are ignored, when considering unsteady processes, remains valid.

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REFERENCES

1. KULIKOVSKII, A. G. and SVESHNIKOVA, E. I., The self-similar problem of the action of an abrupt loading on the boundary of an elastic half space. *Prikl. Mat. Mekh.*, 1985, **49**, 2, 284–291.
2. KULIKOVSKII, A. G. and SVESHNIKOVA, E. I., The decomposition of an arbitrary initial discontinuity in an elastic medium. *Prikl. Mat. Mekh.*, 1988, **52**, 6, 1007–1012.
3. KULIKOVSKII, A. G. and SVESHNIKOVA, E. I., *Non-linear Waves in Elastic Media*. Mosk. Litsei, Moscow, 1998.
4. KULIKOVSKII, A. G. and CHUGAINOVA, A. P., The stability of quasitransverse shock waves in anisotropic elastic media. *Prikl. Mat. Mekh.*, 2000, **64**, 6, 1020–1026.
5. SEDOV, L. S., *Continuous Mechanics*, Vol. 2. Nauka, Moscow, 1984.
6. CHUGAINOVA, A. P., The formation of a self-similar solution in the problem of non-linear waves in an elastic half-space. *Prikl. Mat. Mekh.*, 1988, **52**, 4, 692–697.
7. KULIKOVSKII, A. G. and CHUGAINOVA, A. P., The conditions for the decomposition of a non-linear wave in a viscoelastic medium. *Zh. Vychisl. Mat. Mat. Fiz.*, 1998, **38**, 2, 315–323.
8. KULIKOVSKII, A. G., The equations describing the propagation of non-linear quasitransverse waves in a weakly anisotropic elastic solid. *Prikl. Mat. Mekh.*, 1986, **50**, 4, 597–604.
9. KADOMTSEV, B. B. and PETVIASHVILI, V. I., The stability of solitary waves in weakly dispersive media. *Dokl. Akad. Nauk SSSR*, 1970, **192**, 4, 753–756.
10. ZABOLOTSKAYA, Ye. A. and KHOKHLOV, R. V., Quasiplane waves in the non-linear acoustics of bounded beams. *Akust. Zh.*, 1969, **15**, 1, 40–47.
11. SAMARSKII, A. A. and POPOV, Yu. P., *Difference Methods for Solving Problems in Gas Dynamics*. Nauka, Moscow, 1980.

Translated by E.L.S.